COMPLEX DYNAMIC PHENOMENA IN
ENVIRONMENTAL PLANNING AND MANAGEMENT

Sergio Rinaldi, DEI, Politecnico di Milano, Italy

ENS – Paris – France – 29-30 Avril 2004

1. ENVIROMENTAL MANAGEMENT AND NONLINEAR DYNAMICS
   An overwiew of the most typical problems one encounters in enviromental planning and management. Emphasis on relationships with nonlinear dynamics.Further reading: *Journal of Enviromental Management* (1996), 48, 357-373.

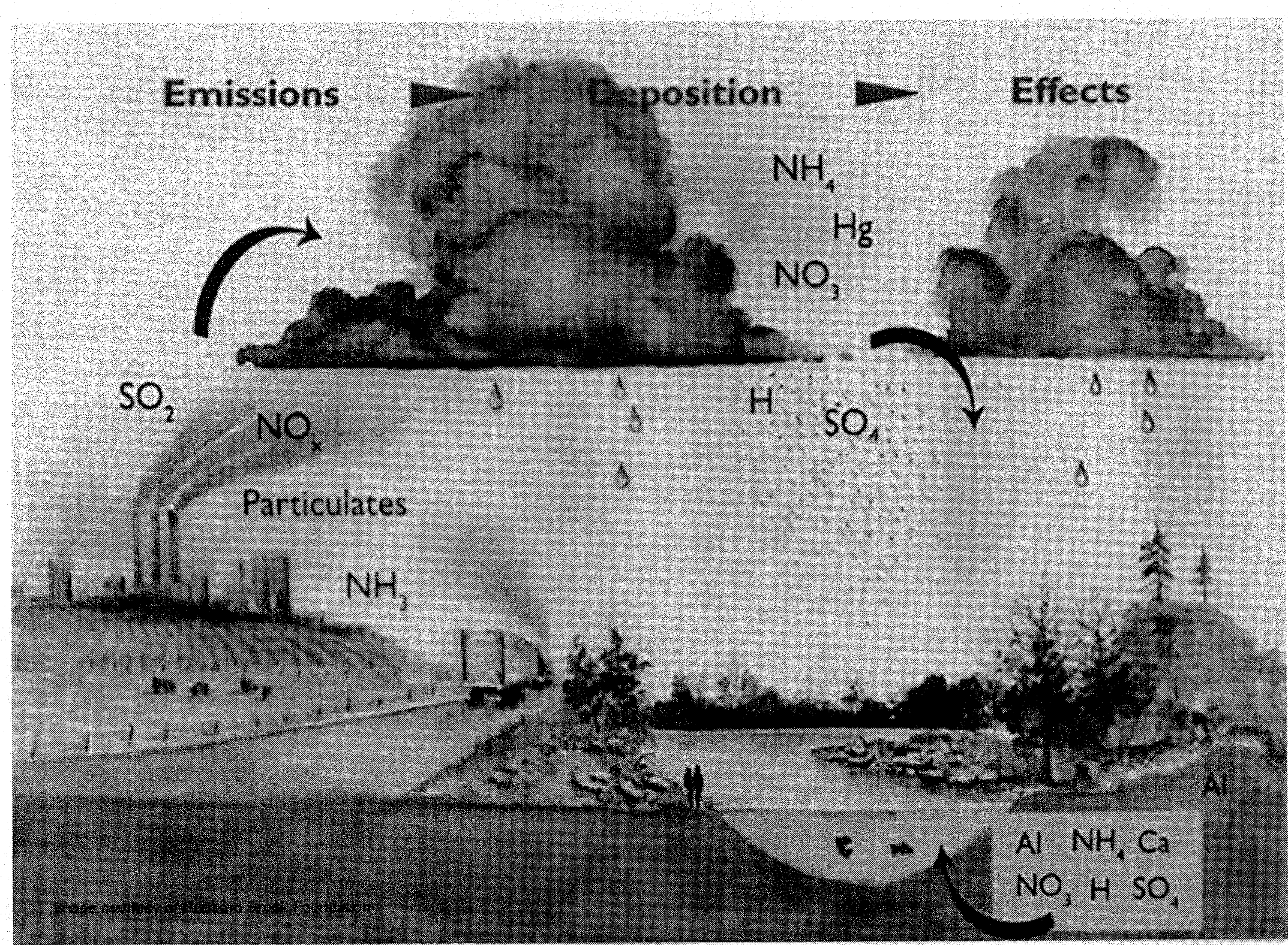
2. THE PROBLEM OF FLOATING PLANTS IN RESERVOIRS
   Description of the problem through a model of competition between floating and submerged plants. Analysis of the model: alternative stable states. Bifurcation analysis and derivation of possible control actions. Analysys of the history of Lake Kariba on the Zambesi river.
   Further raeading: *PNAS* (2003), 100, 4040-4045.

3. FOREST EXPLOITATION AND ACID RAIN: A DANGEROUS MIX
   Description of the problem through a series of minimal models. Existence of catastrophic bifurcations (forest collapse). Cusp bifurcation: negative synergistic effect of acid rain and exploitation.
   Further reading: *Vegetatio* (1987), 69, 213-222
   Appl. Math. Modelling (1989), 13, 674-681
   Theor. Pop. Biol. (1998), 54, 257-269.

4. THE RECLAMATION OF EUTROPHIC WATER BODIES
   Description of the problem in terms of minimal models involving algae, zooplankton and planktivorous fish. Analysis of the bifurcations of the model: the appearance and disappearance of clear-water regimes. Biological control.
   Further reading: *OIKOS* (1997), 80, 519-532.

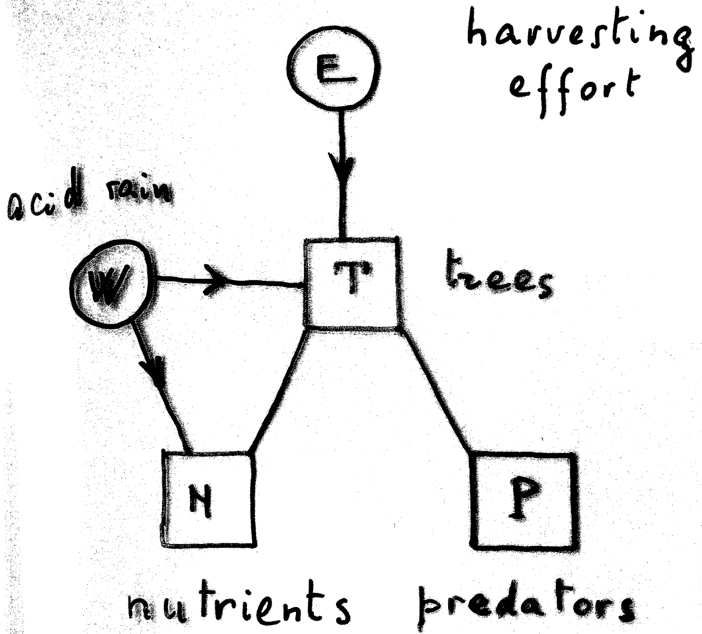
5. TOURISM SUSTAINABILITY: AN OVERVIEW
   The three components of the problem: tourists, environment and facilities. Detection of possible scenarios. Profitable, compatible and sustainable policies. Adaptivity. The case of alternative classes of tourists and of diversified investments.
   Further reading: *Conservation Ecology* (2002), 6(1): 13 [online].
   Chaos and Complexity Letters (2004) first issue (in the press).

6. ENRICHMENT AND YIELD MAXIMIZATION
   Exploitation of renewable resources. Enrichment and mean yield maximization. Analysis of the case of tritrophic food chains. Optimality at the edge of chaos. Derivation of management rules.
   Further reading: *Am. Nat.* (1997) 150, 328-345
   Bull. Math. Biol. (1998) 60, 703-719
   Ecol. Lett. (1999) 2, 6-10
   J. Math. Biol. (2002) 45, 396-418.

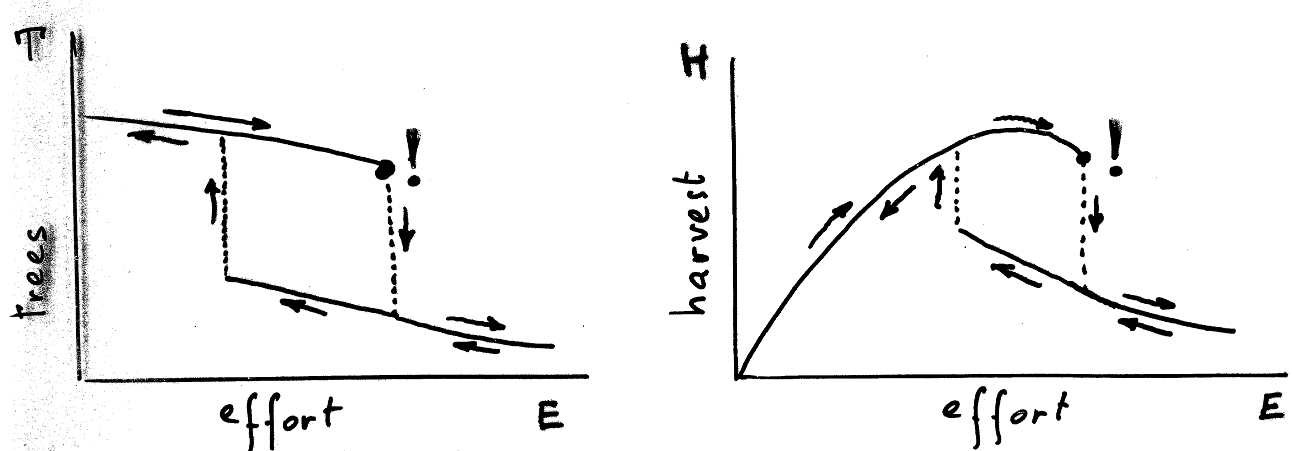


Emissions
Effects
NH_4
Hg
NO_3
SO_2
NO_x
H
SO_4
Particulates
NH_3
Al
Al NH_4 Ca
NO_3 H SO_4

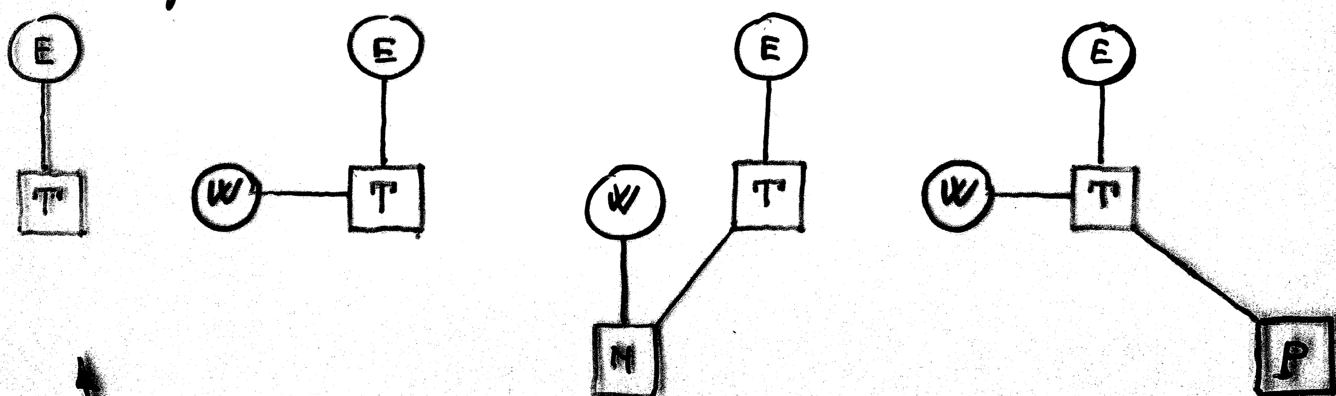
parameters



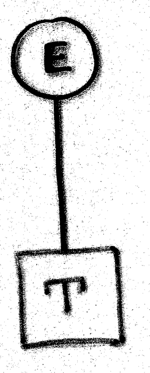
dynamical systems



and similarly for W

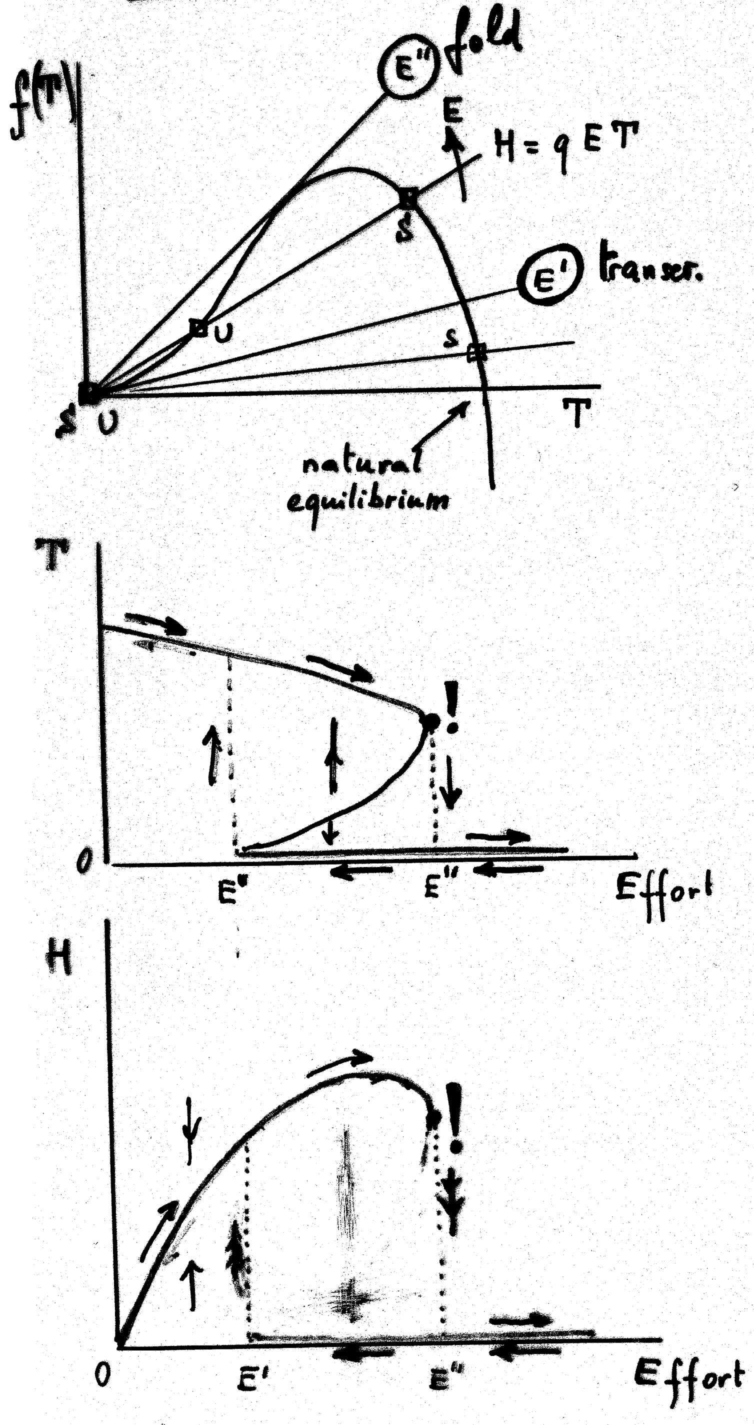


in this lecture

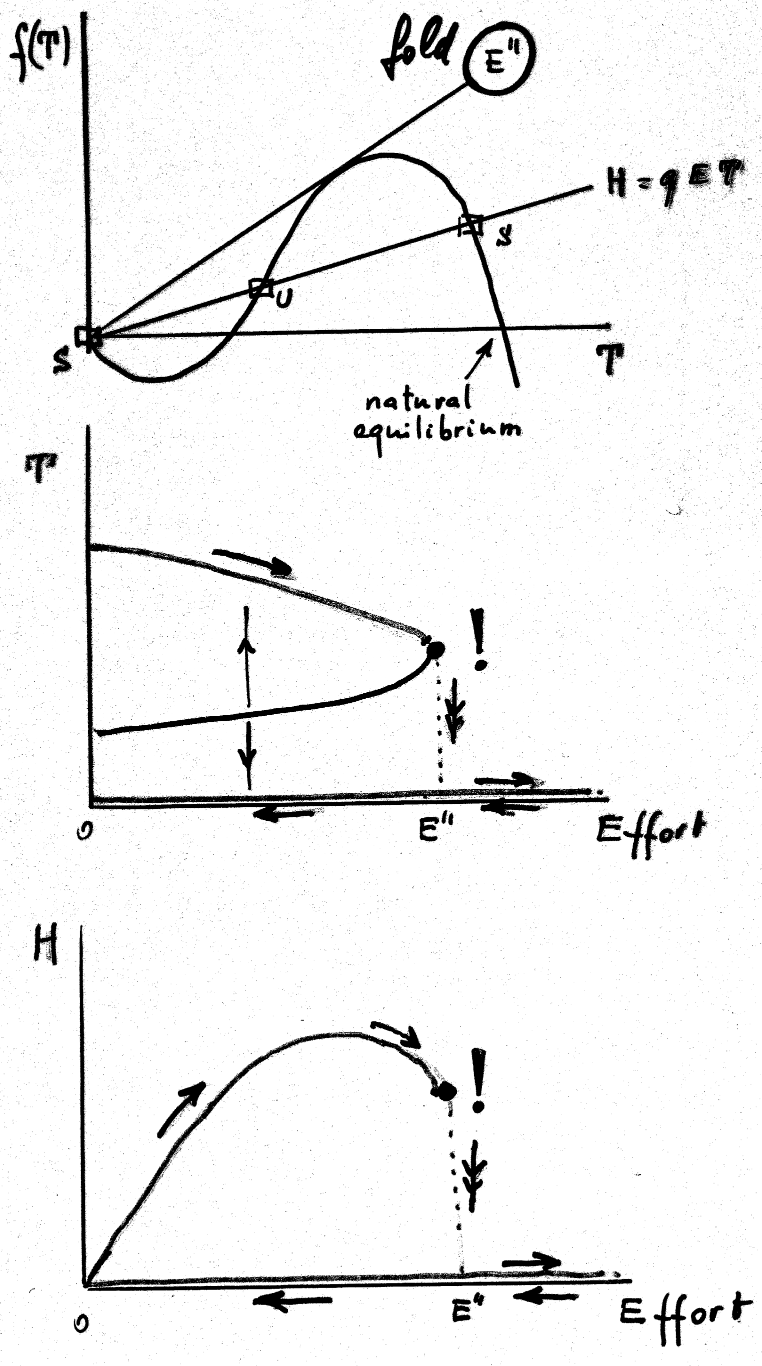


$$\dot{T} = f(T) - q\,E\,T$$

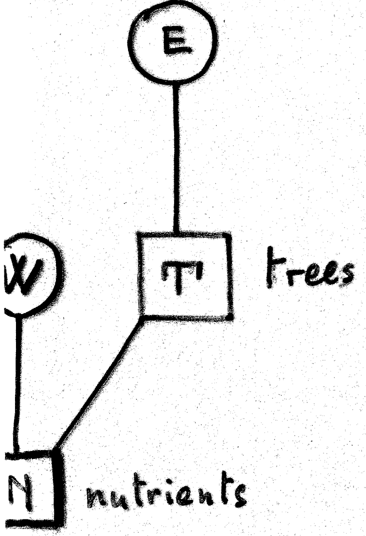
harvest rate H



reversible catastr.

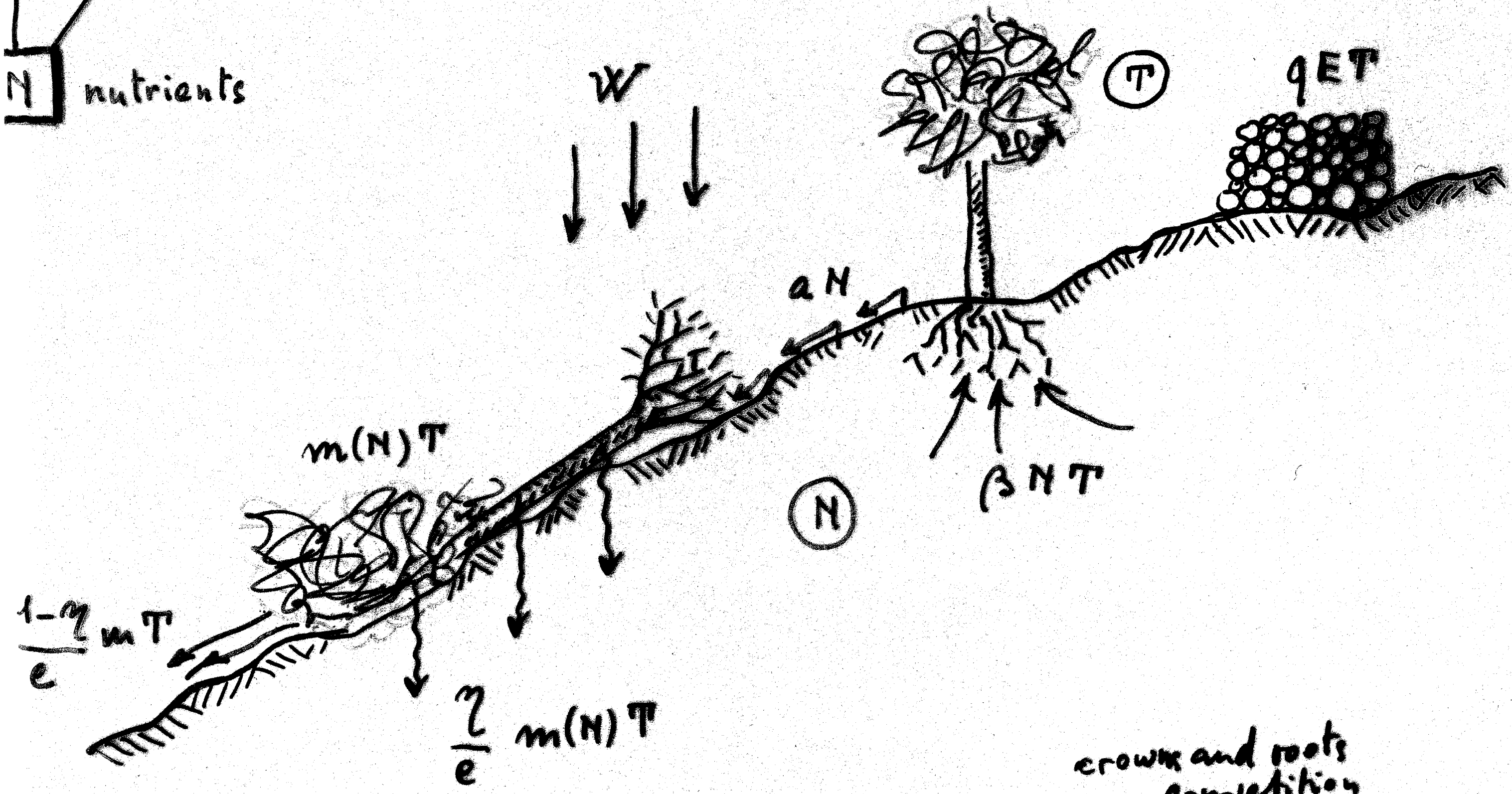


irreversible catastr.



$$\dot{T} = \ ?$$

$$\dot{N} = \ ?$$

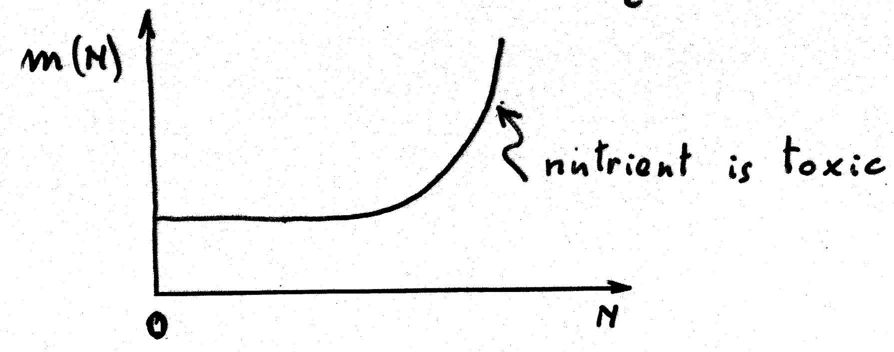


crowns and roots competition

$$\dot{T} = \left(-m(N) + e\beta N\right)T - \overbrace{\underset{1}{q}ET}^{\text{harvest}} - dT^2$$

$$\dot{N} = W - aN - \beta NT + \underset{c}{\frac{\eta}{e}}\, m(N)\, T$$

$\beta \Rightarrow b$ in the paper



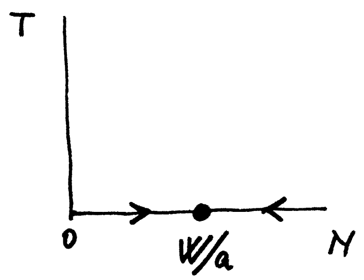
ANALYSIS OF THE MODEL

$$\dot{N} = W - aN - bNT + c\,m(N)T$$

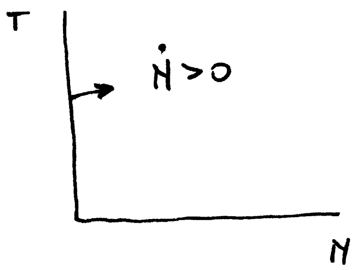
$$\dot{T} = \left(ebN - dT - m(N) - E\right)T$$

- <u>The model is positive</u>

$$\begin{cases} T = 0 \Rightarrow \dot{T} = 0 \Rightarrow T = \text{const.} \Rightarrow T \equiv 0 & \text{i.e. } T \text{ remains equal to zero} \\ \text{the } N \text{ axis is invariant} \end{cases}$$



$N = 0 \Rightarrow \quad \dot{N} = W + c\,m(0)T > 0 \quad$ i.e. N becomes positive



- <u>There are no cycles</u> (never been proved)

- <u>There can be multiple equilibria</u>

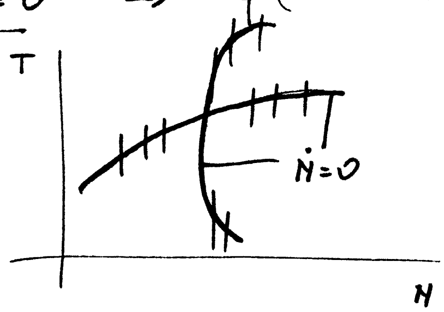
- <u>There are transcritical and fold bifurcations</u>

The last two points will be discussed graphically

EQUILIBRIA AND ISOCLINES

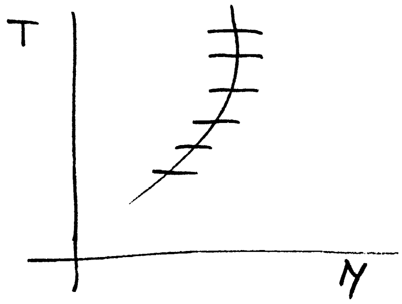
$$\dot{N} = f(N\,T\,W)$$
$$\dot{T} = g(N\,T\,E)$$

Isocline $\dot{N} = 0 \Rightarrow f(N\,T\,W) = 0$



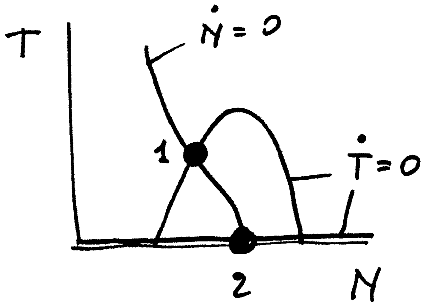
On the isoclines $\dot{N} = 0$ the trajectory is vertical

Isocline $\dot{T} = 0 \Rightarrow g(N\,T\,E) = 0$



On the isoclines $\dot{T} = 0$ the trajectory is horizontal

Equilibria are a the intersections of the isoclines



The stability of the equilibria can be studied through linearization (or intuitively!)

Remark The isoclines $\dot{N} = 0$ vary with the parameter W

The isoclines $\dot{T} = 0$ vary with the parameter E

ISOCLINES

$\dot{N} = 0$ (W)

$$W - aN - bNT + c\,m(N)\,T = 0$$

$$T = \frac{1}{c}\,\frac{W - aN}{\frac{b}{c}N - m(N)}$$

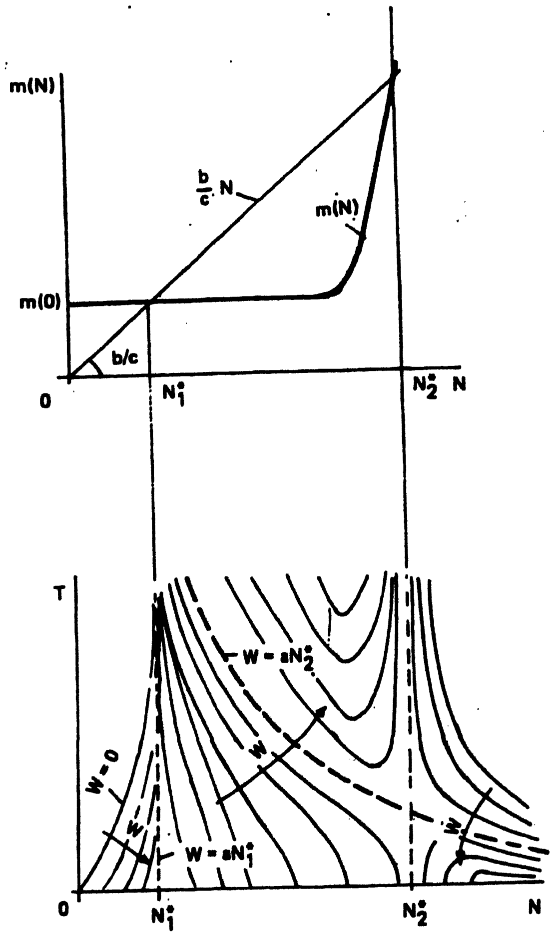
$T = 0$ for $N = \frac{W}{a}$

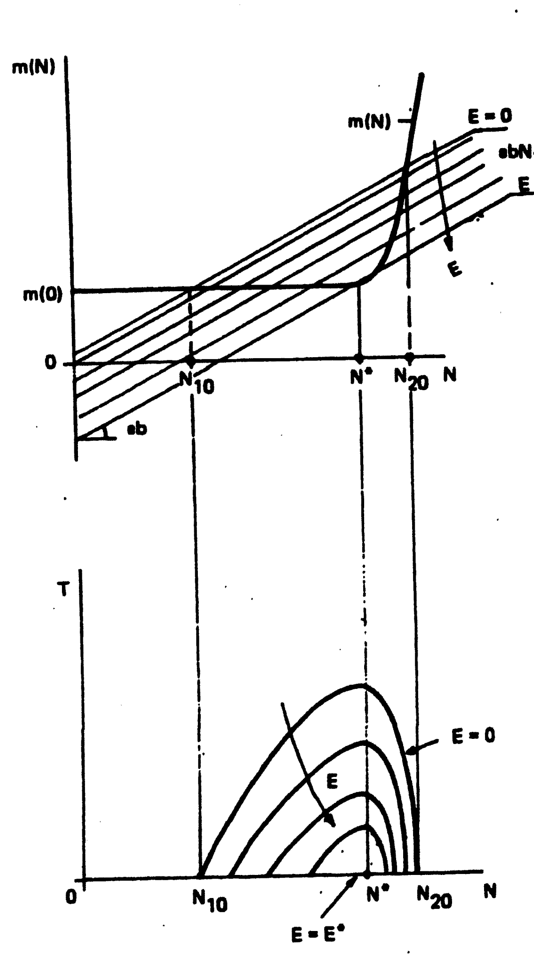
$T = \infty$ for $m(N) = \frac{b}{c}N$

$\dot{T} = 0$ (E)

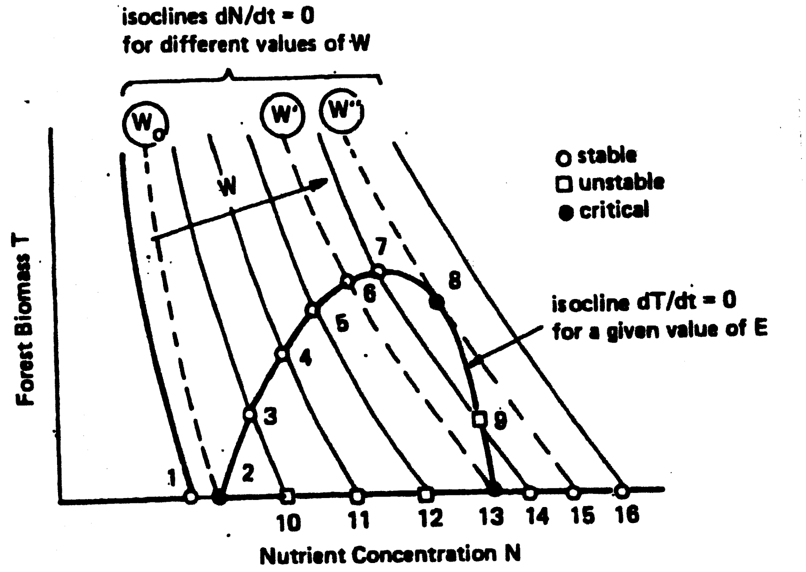
$$\begin{cases} T = 0 \\ T = \frac{1}{d}\left[ebN - m(N) - E\right] \end{cases}$$

two branches :
one is the N axis
the other is a sort of parabola
with intersections with the N axis
at $m(N) = ebN - E$

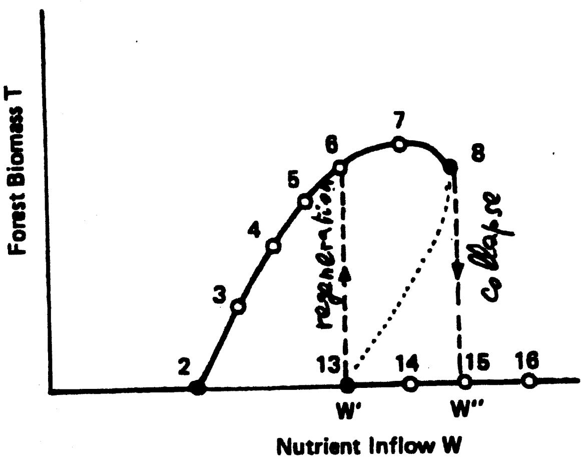




BIFURCATIONS WITH RESPECT TO W



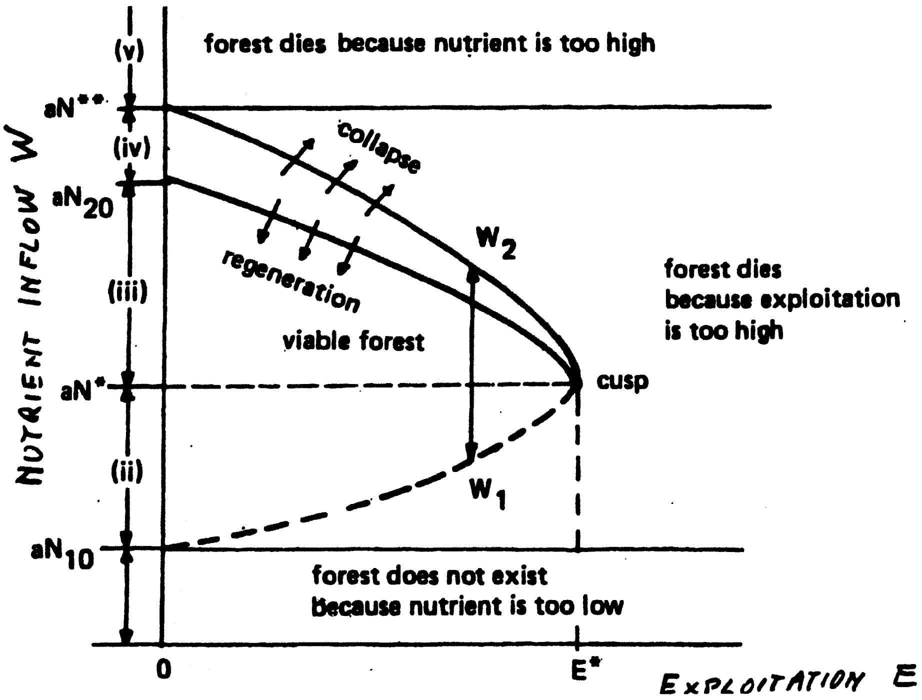
E is fixed



8 = fold bifurcation

13 = transcr. bifurc.

2 = transcr. bifurc.

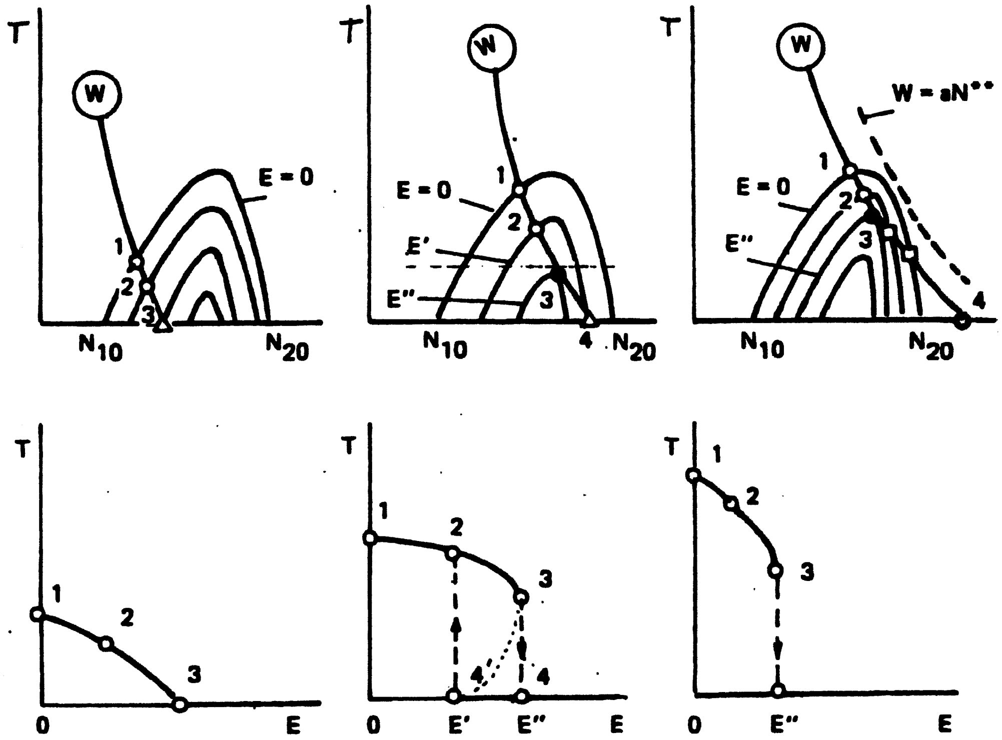


8 = collapse

13 = regeneration

2 = smooth transition to death (- - - - -

BIFURCATIONS WITH RESPECT TO E



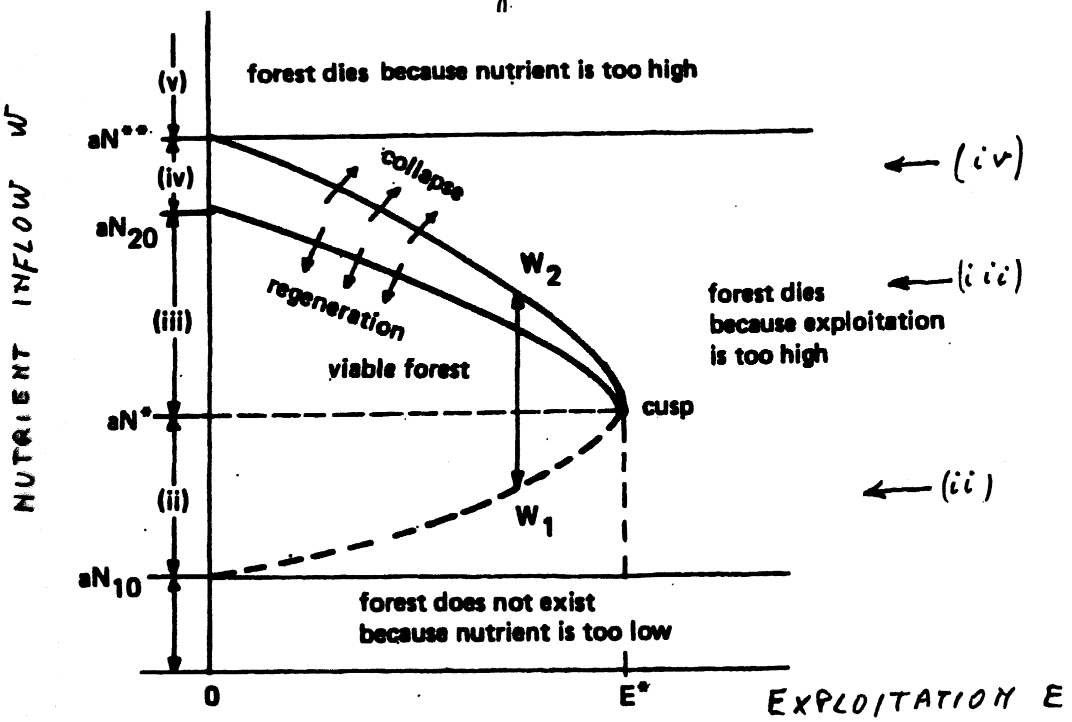
Case (ii) Case (iii) Case (iv)

W = fixed
(at different values)

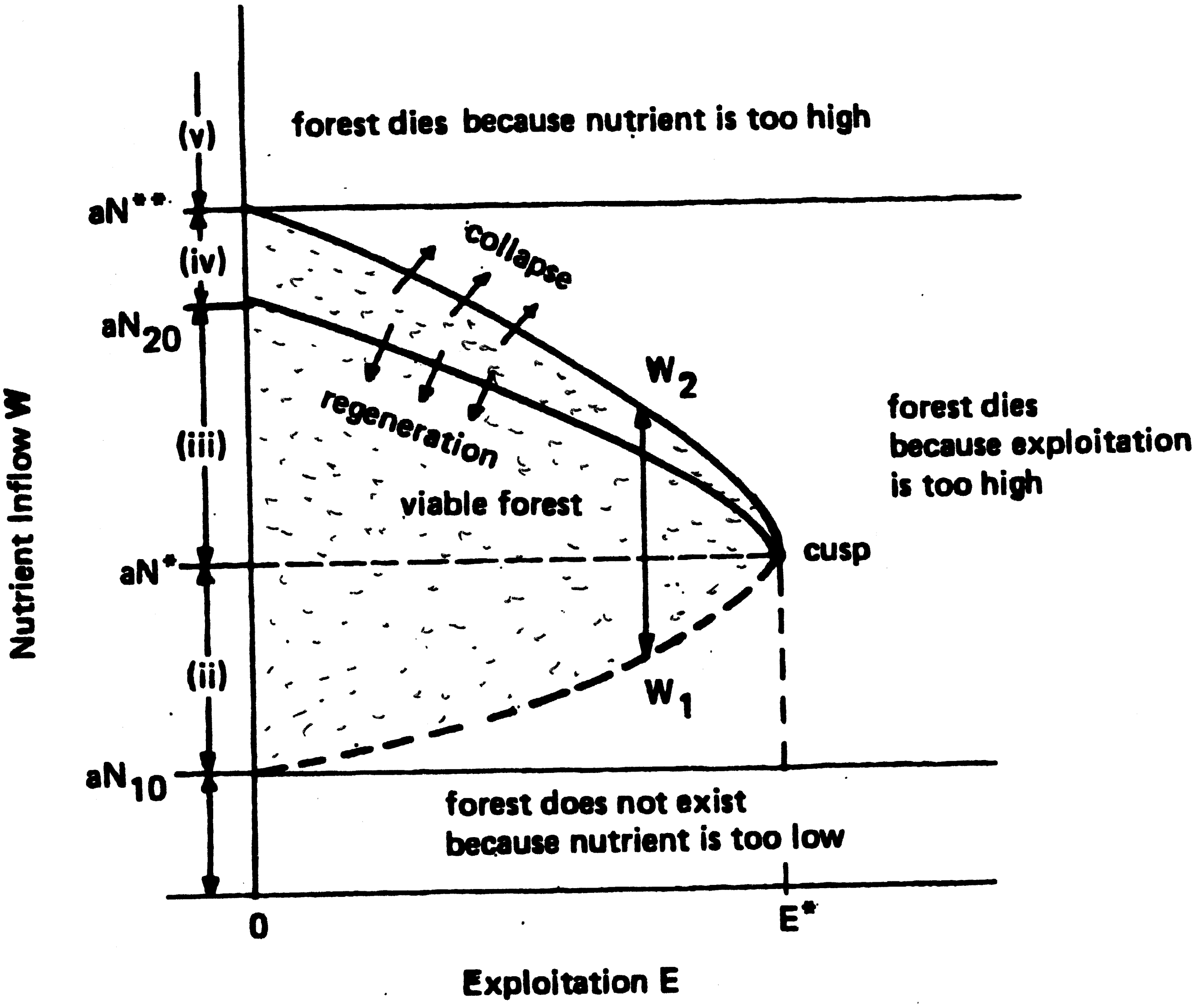
3 = transcrit.
smooth transition to death

3 = fold
4' = transcrit.
collapse and regeneration

3 = fold
collapse
no regeneration



SUMMARY OF RESULTS



Exploited forests are more fragile w.r.t. acid rain
Forests exposed to acid rain are more fragile w.r.t. exploitat[ion]

The cusp is a codimension-2 bifurcation point because at the cusp there are two independent degeneracies: