

COMPLEX DYNAMIC PHENOMENA IN  
ENVIRONMENTAL PLANNING AND MANAGEMENT  
Sergio Rinaldi, DEI, Politecnico di Milano, Italy  
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1. ENVIRONMENTAL MANAGEMENT AND NONLINEAR DYNAMICS

An overview of the most typical problems one encounters in environmental planning and management. Emphasis on relationships with nonlinear dynamics. Further reading: *Journal of Environmental Management* (1996), 48, 357-373.

2. THE PROBLEM OF FLOATING PLANTS IN RESERVOIRS

Description of the problem through a model of competition between floating and submerged plants. Analysis of the model: alternative stable states. Bifurcation analysis and derivation of possible control actions. Analysis of the history of Lake Kariba on the Zambesi river. Further reading: *PNAS* (2003), 100, 4040-4045.

3. FOREST EXPLOITATION AND ACID RAIN: A DANGEROUS MIX

Description of the problem through a series of minimal models. Existence of catastrophic bifurcations (forest collapse). Cusp bifurcation: negative synergistic effect of acid rain and exploitation.

Further reading: *Vegetatio* (1987), 69, 213-222  
*Appl. Math. Modelling* (1989), 13, 674-681  
*Theor. Pop. Biol.* (1998), 54, 257-269.

4. THE RECLAMATION OF EUTROPHIC WATER BODIES

Description of the problem in terms of minimal models involving algae, zooplankton and planktivorous fish. Analysis of the bifurcations of the model: the appearance and disappearance of clear-water regimes. Biological control.

Further reading: *OIKOS* (1997), 80, 519-532.

5. TOURISM SUSTAINABILITY: AN OVERVIEW

The three components of the problem: tourists, environment and facilities. Detection of possible scenarios. Profitable, compatible and sustainable policies. Adaptivity. The case of alternative classes of tourists and of diversified investments.

Further reading: *Conservation Ecology* (2002), 6(1): 13 [online].  
*Chaos and Complexity Letters* (2004) first issue (in the press).

6. ENRICHMENT AND YIELD MAXIMIZATION

Exploitation of renewable resources. Enrichment and mean yield maximization. Analysis of the case of tritrophic food chains. Optimality at the edge of chaos. Derivation of management rules.

Further reading: *Am. Nat.* (1997) 150, 328-345  
*Bull. Math. Biol.* (1998) 60, 703-719  
*Ecol. Lett.* (1999) 2, 6-10  
*J. Math. Biol.* (2002) 45, 396-418.

# FLOATING PLANTS DOMINANCE

Ponds  
ditches  
tropical lakes  
reservoirs

} are sometimes dominated by  
floating plants

*Eichhornia crassipes*

*Pistia stratiotes*

*Salvinia molesta*

duckweeds (Lemnaceae)

water ferns (Azollaceae)

Floating plants dominance  $\Rightarrow$  problems in navigation  
and recreation

$\Downarrow$   
anoxic conditions

$\Downarrow$   
reduction of animal biomass  
" " biodiversity

$\Downarrow$   
negative impact on fisheries

Routed submerged plants dominance

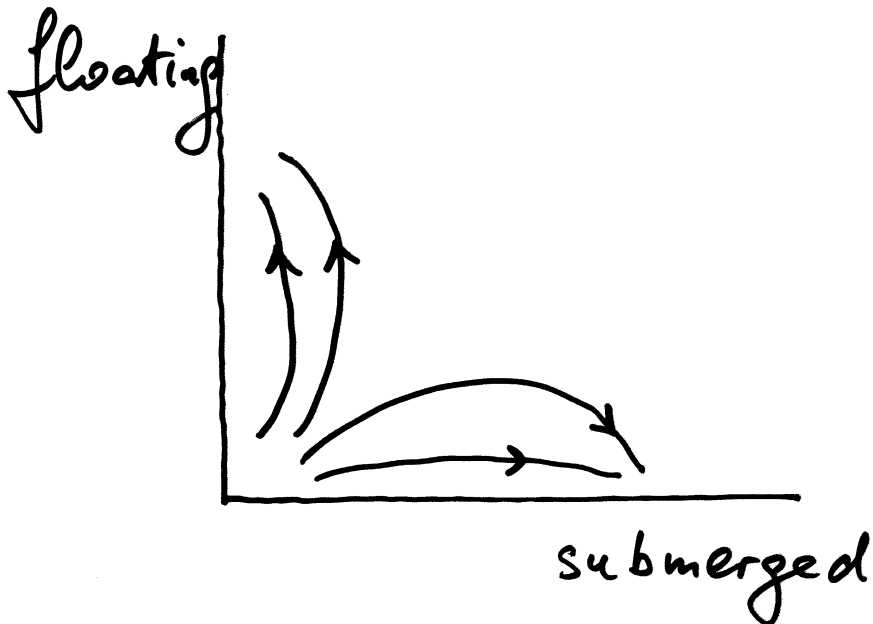
$\Downarrow$   
much better situation

Fact 1: floating and submerged plants compete for  
nutrients and light

Fact 2: competition  $\Rightarrow$  alternative stable states (bistability)

# Outline of the lecture

1. Evidence of alternative stable states (see figure)
  - 1a. Laboratory experiments
  - 1b. Analysis of field data
  - 1c. Analysis of a minimal model
  - 1d. Analysis of a complex simulation model
2. Remedies (mainly from the model)
3. History of Lake Kariba



# 1a. Laboratory experiments

submerged plant : *Elodea nuttallii*

floating duckweed : *Lemna gibba*

competition for 57 days in 8 liter containers

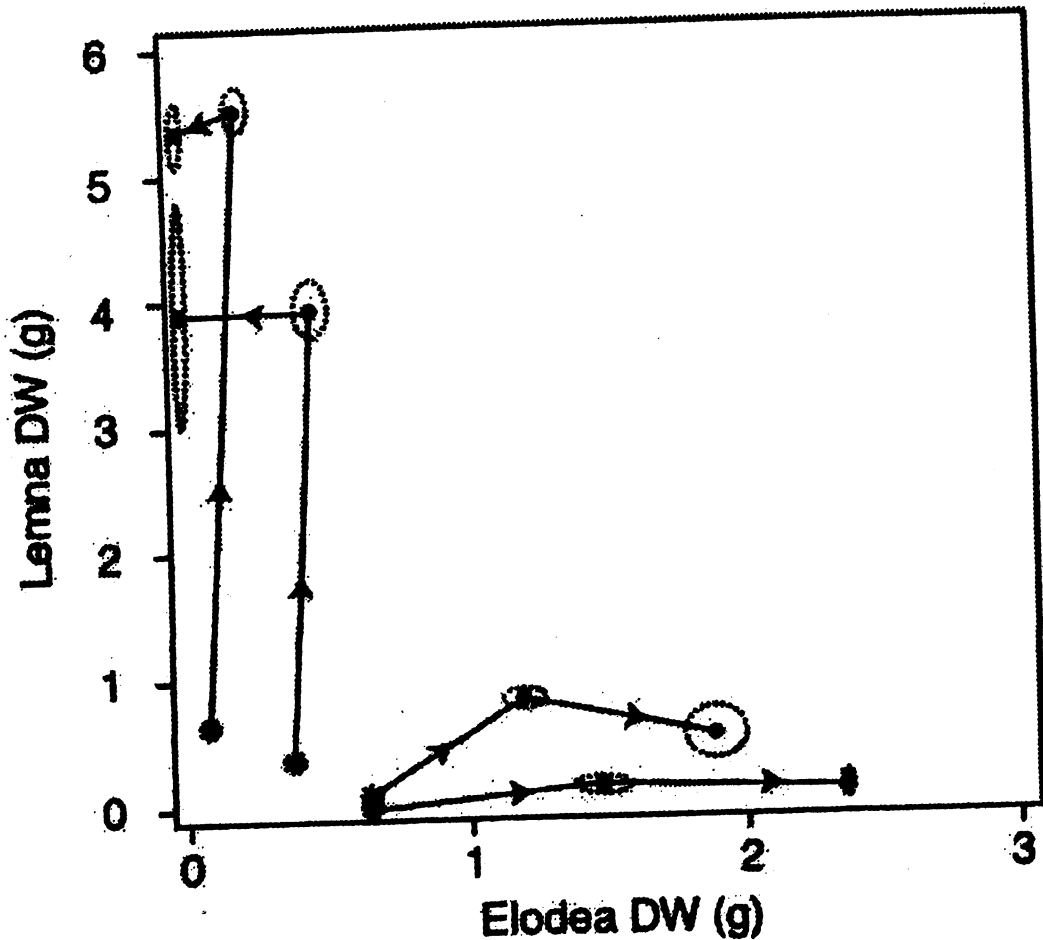
water temperature 23-25 °C

light : 16 h dark / 8 h light

samples : 1st, 23rd, 57th day

initial conditions : 4

two or more replicates

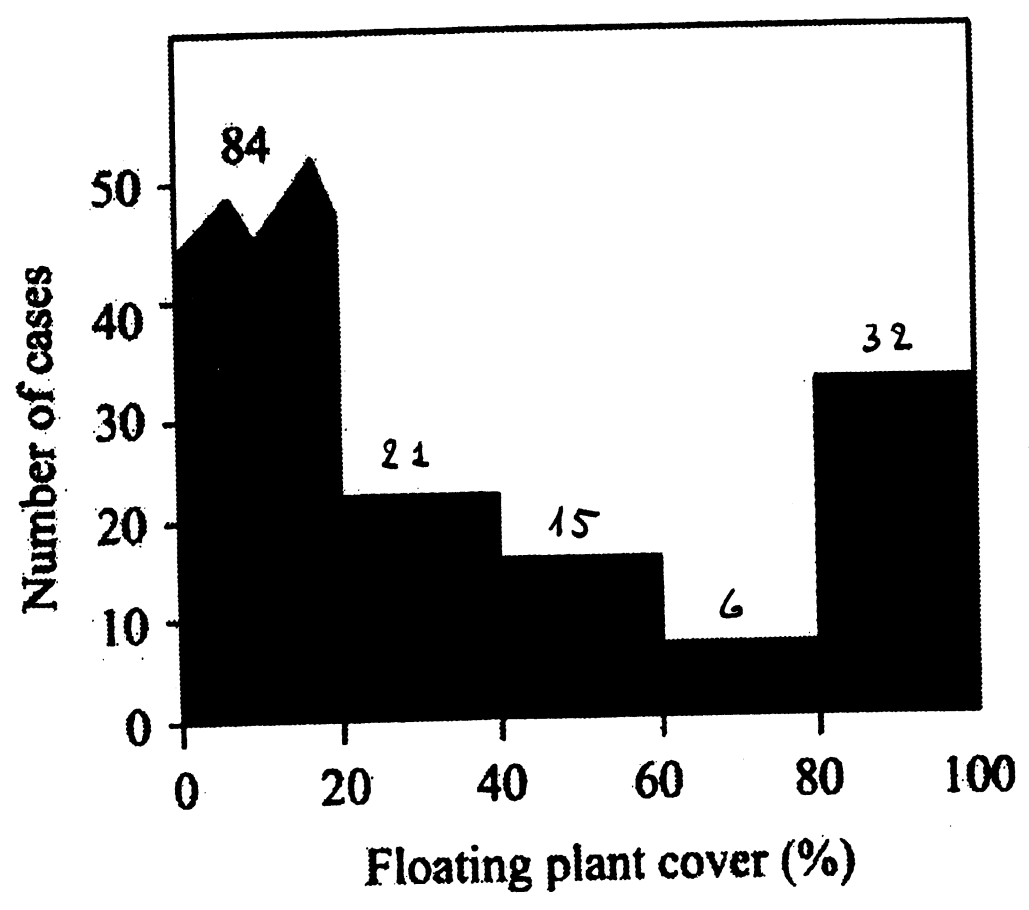


# 1 b. Field data

data set : vegetation censuses of 641 Dutch ditches

more precisely : 158 ditches with total vegetation cover > 80%

result : bimodal distribution



1c. A minimal model : assumptions

minimal = "minimum" number of variables



S = submerged      F = floating

$$\dot{S} = r_s S \frac{n}{n + h_s} \frac{I}{1 + a_s S + b F + W} - l_s S$$

$$\dot{F} = r_f F \frac{n}{n + h_f} \frac{I}{1 + a_f F} - l_f F$$

$r$  = maximum growth rate per capita

$l$  = losses

$h$  = half-saturation concentration

$1/a$ ,  $1/b$  = densities of 50% reduction due to competition for light

nutrient concentration  $n = \frac{N}{1 + q_s S + q_f F}$

$N \equiv$  nutrient load

Variable	Value	Units
F	-	g dw m <sup>-2</sup>
S	-	g dw m <sup>-2</sup>
N	-	mg N liter <sup>-1</sup>
n	-	mg N liter <sup>-1</sup>
$a_f$	0.01	(g dw m <sup>-2</sup> ) <sup>-1</sup>
$a_s$	0.01	(g dw m <sup>-2</sup> ) <sup>-1</sup>
b	0.02	(g dw m <sup>-2</sup> ) <sup>-1</sup>
$h_f$	0.2	mg N liter <sup>-1</sup>
$h_s$	0.0	mg N liter <sup>-1</sup>
$l_f$	0.05	day <sup>-1</sup>
$l_s$	0.05	day <sup>-1</sup>
$q_f$	0.005	(g dw m <sup>-2</sup> ) <sup>-1</sup>
$q_s$	0.075	(g dw m <sup>-2</sup> ) <sup>-1</sup>
$r_f$	0.5	day <sup>-1</sup>
$r_s$	0	-
$W$	0	-
$r_f$	0.5	day <sup>-1</sup>

$b > a_s$  cross-shading > self-shading  
 $h_f > h_s$  submerged do not need nutrient in the water column  
 $q_s > q_f$  submerged have a greater impact on nutrient  
 $r_s > l$  plants invade

1 c. A minimal model : properties

$$\begin{cases} \dot{S} = r_s S \frac{n}{n+h_s} \frac{1}{1+a_s S + b F + W} - l_s S \\ \dot{F} = r_f F \frac{n}{n+h_f} \frac{1}{1+a_f F} - l_f F \end{cases}$$

$$n = \frac{N}{1+q_s S + q_f F}$$

(i) the model is positive

$$S(0) \geq 0, F(0) \geq 0 \Rightarrow S(t) \geq 0, F(t) \geq 0 \quad \forall t$$



$$F(0) = 0 \Rightarrow \dot{F}(0) = 0$$

↓

the S axis is invariant  
(same for F axis)

(ii) There are no limit cycles

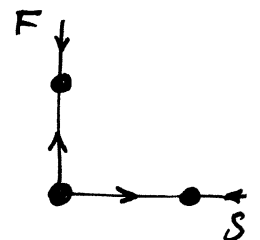
This property is not easy to prove (Bendixon-Poincaré theory)

This property implies that attractors, repellors and saddles can only be equilibria

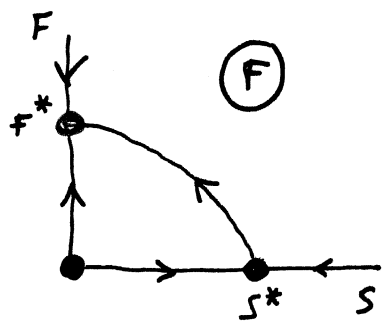
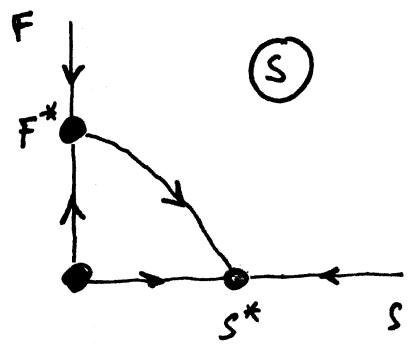
(i) + (ii)  $\Rightarrow$  look for constant solutions (equilibria) in the positive quadrant

(iii) the origin is always an equilibrium

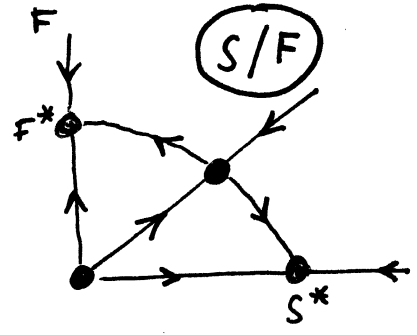
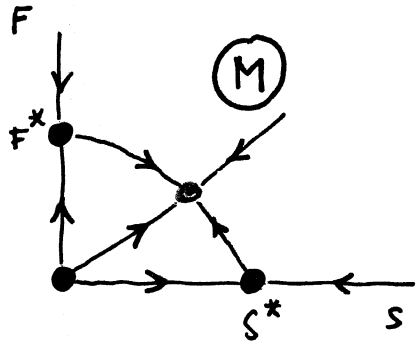
(iv) there is always an equilibrium  $(S^*, 0)$  and an equilibrium  $(0, F^*)$



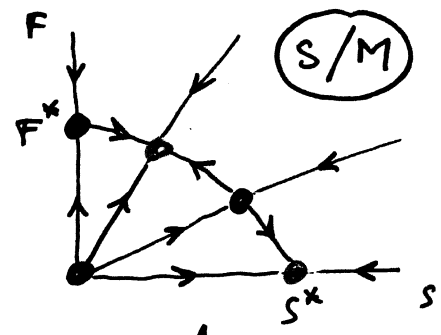
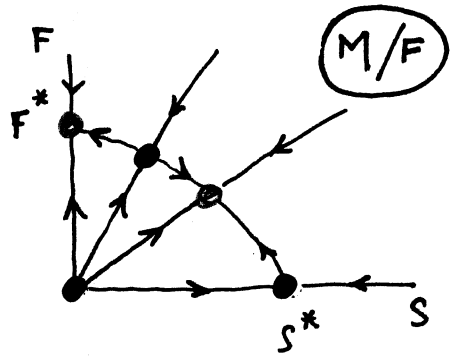
1 c. A minimal model : possible state portraits



no strictly positive equilibria  
(why  $F^*$  and  $S^*$  cannot be both stable or unstable?)  
↙



one strictly positive equilibrium



two strictly positive equilibria (one stab. and one unstable)

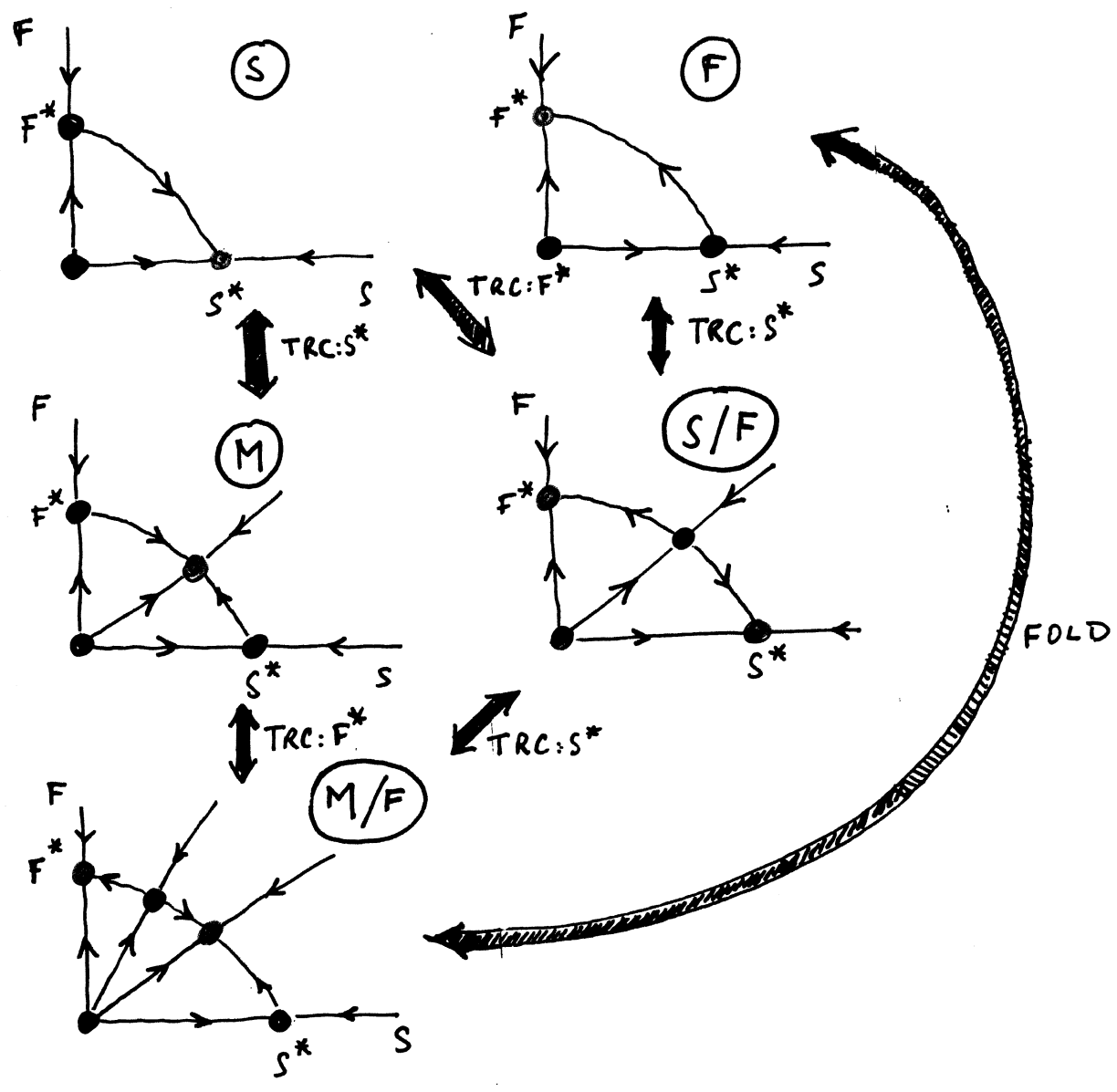
due to the numerical values of the parameters this case does not exist

Bifurcations : transitions from one state portrait to another due to variations of parameters

Remark : in this case bifurcations must be collisions of equilibria



1 c. A minimal model : bifurcations



Transcritical TRC : two equilibria collide and exchange stability

Fold : two equilibria collide and disappear

Remarks : □ transcritical bifurcations are generic in positive systems

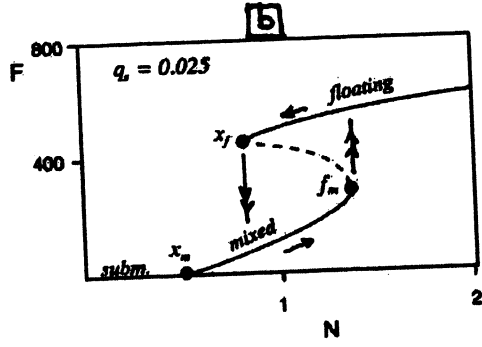
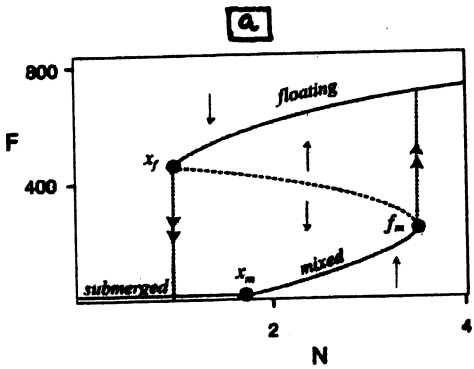
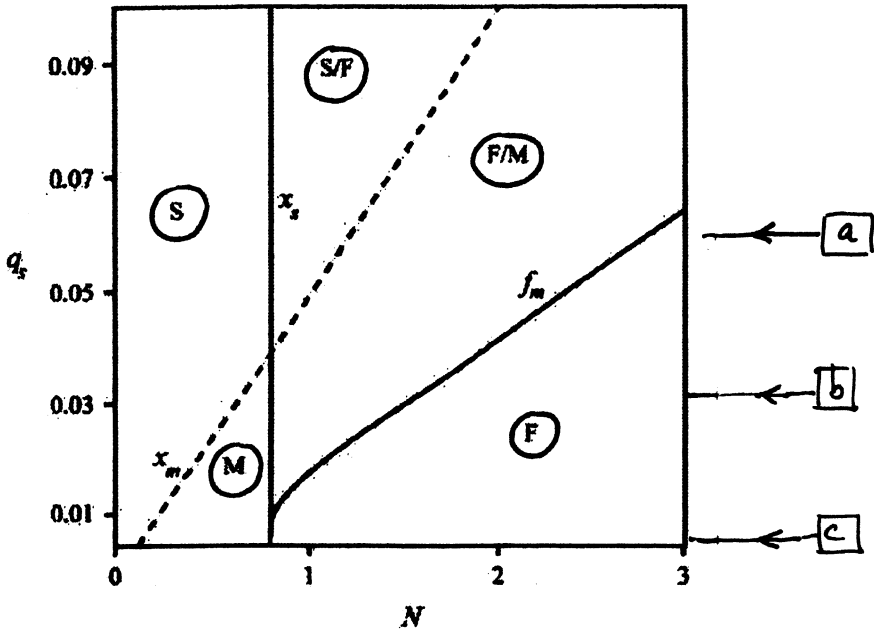
- $F/S \rightarrow S$  is a "good" catastrophic bifurcation
- $S/F \rightarrow F$  is a "bad" catastrophic bifurcation
- $M/F \rightarrow F$  is a "bad" catastrophic bifurcation

1c. A minimal model : bifurcation diagram

Fix all parameters but two :  $N$  and  $q_s$

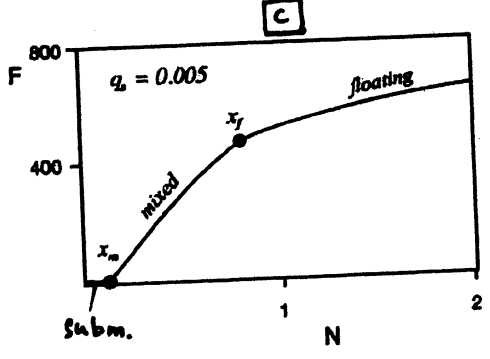
↑  
nutrient load

↑  
impact of submerged plants on nutrient



hysteresis

harvest floating plants to switch to the alternative attractor



$a + b \Rightarrow$  decrease  $N$  to eliminate floating plant

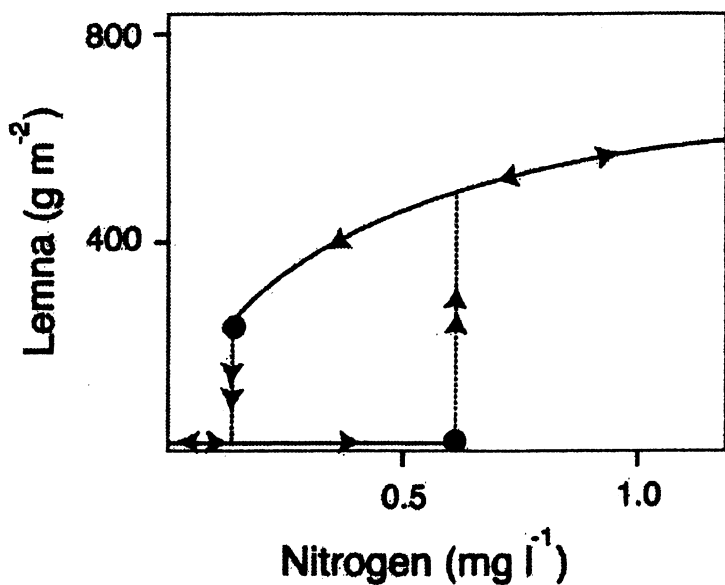
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a very small increase of  $N$  can trigger a switch to floating plant

# 1 d. Analysis of a complex simulation model

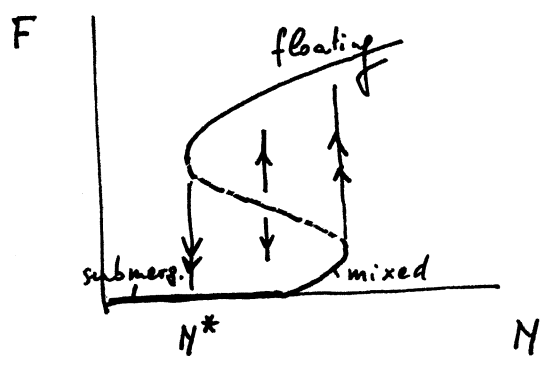
complex model with much more details to check if the results of the minimal model are robust

- spatially explicit
- individual based
- seasons
- dormant stages in winter
- daily cycle for light
- age of plants



See Aquat. Bot. 45, 341-356 (1993) for more details

## 2. REMEDIES



hysteresis (see page 9)

first remedy : decrease  $N$  below  $N^*$  to force the transition floating  $\rightarrow$  submerged  
 This can be done through "treatment"  
 It is often costly and long to obtain satisfactory results

second remedy harvest floating plants, so that  $F$  is reduced below the unstable equilibrium, thus triggering a transition to the "submerged" or "mixed" state

The two remedies can be combined -

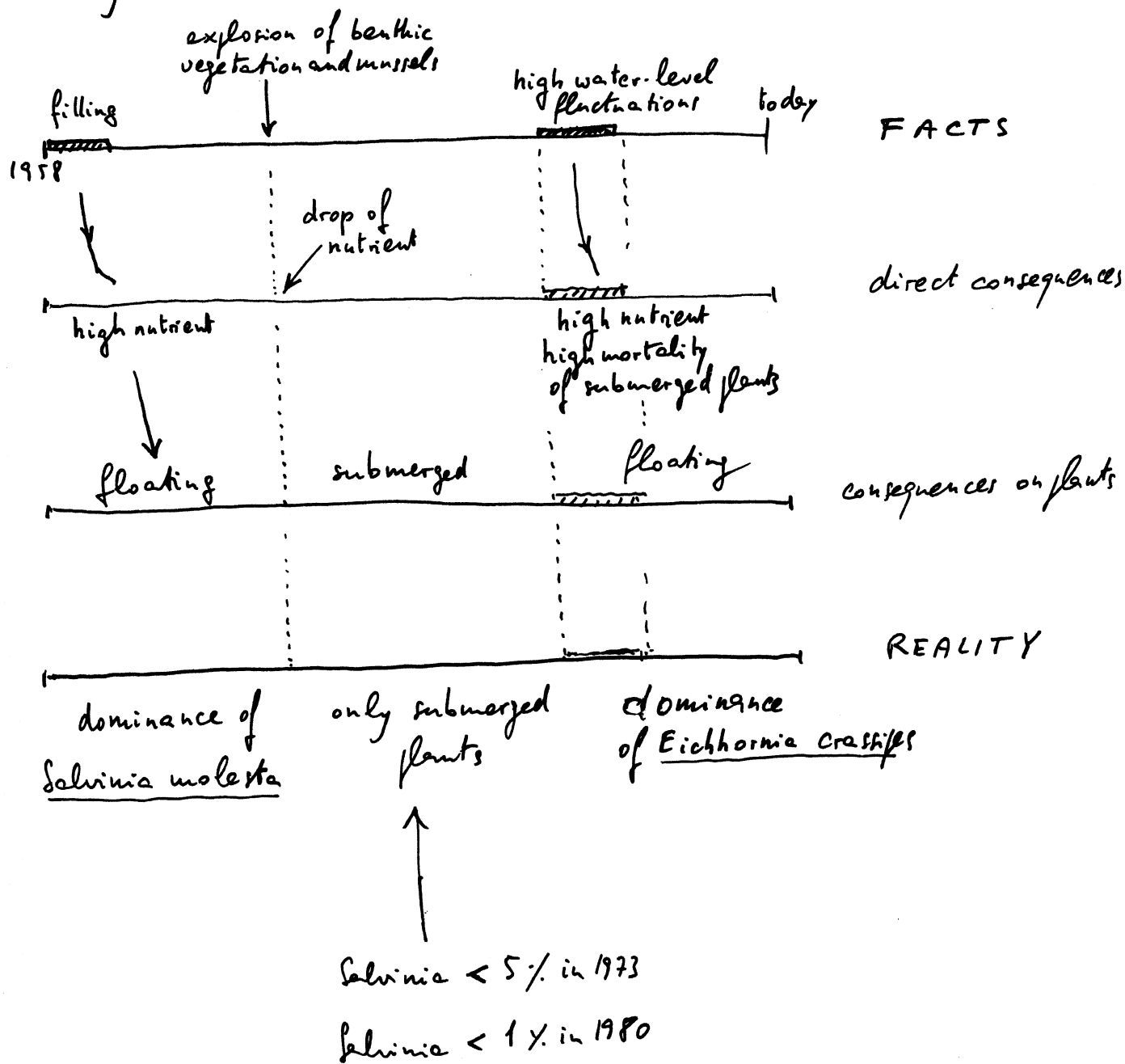
Warning : If the water body is in the "mixed" state, control the nutrient load so that it does not increase. This will avoid the transition to the "floating" state.

### 3. HISTORY OF LAKE KARIBA

Largest manmade African lake

Dam on Zambesi river 1958

History is well documented



Conclusion : the minimal model explains the history of Lake Kariba